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TECHNICAL REPORT 2506

OPTIMUM FRAGMENT DISTRIBUTION FOR AN AIR-GROUND WARHEAD (U)

HENRY DE CICCO

JULY 1958

FELTMAN RESEARCH AND ENGINEERING LABORATORIES
PICATINNY ARSENAL
DOVER, N. J.

FC

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OPTIMUM FRAGMENT DISTRIBUTION FOR AN AIR-GROUND WARHEAD (U)

by

Henry De Cicco

July 1958

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Technical Report 2506

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ABSTRACT

The distribution of fragments that optimizes lethal area for an air-ground warhead is derived for the general case where the probability of incapacitation is variable. A comparison is made for specified conditions between the lethal area related to the optimum distribution (designated ERS-Optimum) and the lethal areas corresponding to the Kent-Hitchcock Contour and a sphere.

~~CONCLUSIONS AND RECOMMENDATIONS~~

~~Equation 13, page 6, gives the~~ A mathematical representation (is given) of the distribution of a fixed number of fragments on the ground that optimizes lethal area. It is shown that the optimum cone angle of dispersion of those fragments is set by that optimum distribution and that this cone angle would only coincide with the terrain limitation under special conditions.

Preliminary indications are that any large departure from the optimum fragment distribution, combined with a disregard for the appropriate cone angle of dispersion, may lead to very considerable losses in realizable lethal area.

It is recommended that the present analysis be extended to include studies of variation of burst height and arbitrary angles of approach, and that the results eventually be considered in the derivation of a specific warhead contour.

INTRODUCTION

1. It has been shown that (certain conditions being fixed) the lethal potential of an air-ground warhead depends on its surface contour, that is, its shape (Ref 1). To a significant extent, the shape of a given warhead determines a distribution of fragments on the ground; and this distribution (again, certain conditions being fixed) in turn determines the average probability of kill over a prescribed ground area.

2. This fact suggests two questions:

a. Given a fixed number of fragments, what distribution of these over a given ground area is optimum in some precise lethality sense?

b. What warhead shape corresponds to this optimum distribution?

3. This report is concerned only with the first of these questions. A subsequent paper will take up the second.

4. Both problems have already received considerable attention (Refs 2 and 3). The conclusions reached indicate that:

a. The optimum distribution of a fixed number of fragments over a given ground area requires that the fragment density, i.e., the number of fragments per unit area, be a constant and

b. The shape of the warhead corresponding to this optimum distribution of fragments is the so-called Kent-Hitchcock contour (which has a certain analytic representation).

5. Previous investigators have assumed, among other things, that the probability of incapacitation, P_{hk}, is constant over a given ground area. In the present paper, that assumption is not made. P_{hk} is treated as a variable so that a distribution of fragments is determined which is optimum in a more general sense.

6. The problem of actually finding the optimum distribution is approached in the present paper as a problem in constrained variation: the lethal potential of a warhead is defined in terms of the lethal area integral, and a distribution function is sought such that the integral is maximized under the condition (constraint) that the number of fragments over a prescribed ground area is constant. The constraining condition itself is stated as a definite integral rather than as an algebraic relation, so that the problem falls into the so-called isoperimetric class.

7. In comparing the lethal area corresponding to the optimum distribution with the lethal areas related to the Kent-Hitchcock Contour and a conventional sphere, the following assumptions have been made:

a. The number of fragments per unit surface area is constant for the entire surface of the exploding warhead and, moreover, all fragments are of the same size.

b. The fragments leave normal to the surface of the warhead.

c. The velocity of the fragments is such that the influence of gravity may be neglected.

d. The initial velocity of all fragments is the same

e. The velocity of the warhead is zero when it explodes.

f. The axis of the warhead is oriented normal to the ground plane.

8. A critical appraisal of these assumptions is given in Reference 1.

DERIVATION OF OPTIMUM FRAGMENT DISTRIBUTION

9. The lethal area of an air-ground warhead is defined as follows:

$$A_L = \iint_{A_T} P_k\,(x, y)\,dx\,dy \tag{1}$$

where P_k (x,y) is the probability density of a kill at (x, y), and A_T is some prescribed ground-area target (thought of as ranging over the xy plane).

10. It can be shown that, for the conditions met in a wide class of lethality problems, the above expression for A_L may be approximated by

$$A_L = \iint_{A_T} [1-\exp(-E_K(x,y))]\,dx\,dy \tag{2}$$

where E_k (x, y) denotes the expected number of disabling hits per human target at (x, y).

11. A more explicit form of Equation 2 is

$$A_L = \iint_{A_T} \left[1-\exp\left(-\frac{f(x, y)\,g(x, y)\,p(x, y)}{\cos\{\tan^{-1}\sqrt{x^2 + y^2}/h\}}\right)\right] dx\,dy \tag{3}$$

where f(x, y) is the density of fragments on the ground at x,y,

$g(x, y)$ is the average presented area of a human target normal to the fragment path at x, y,

$p(x, y)$ is the conditional probability density that a single hit will incapacitate a target at x, y,

A_T is some prescribed circular area on the ground with radius r_o, and

h is the height of burst of the warhead.

12. Insofar as A_T is circular, and P_k depends only on $x^2 + y^2 = r^2$, a transformation to polar coordinates changes Equation 3 to

$$A_L = 2\pi \int_o^{r_o} \left[1 - \exp\left(\frac{-f\, g\, p}{\cos \{\tan^{-1} r/h\}} \right) \right] r\, dr \tag{4}$$

where f, g, and p are now functions of the single variable r (the radius vector).

13. Now the number of fragments from the warhead that will lie in the prescribed ground area is:

$$N = 2\pi \int_o^{r_o} f\, r\, dr \tag{5}$$

14. Then the problem of finding a lethally optimum ground distribution of fragments can be stated in a precise way as follows:

15. Find f such that A_L is a maximum for N fixed. (The function f carries the restriction that it be always non negative.)

16. The solution is made in three steps. In Equation 4, let

$$F = 2\pi r [1 - \exp(-f \sec \{\tan^{-1} r/h\}\, g\, p)] \tag{5a}$$

and in Equation 5 let

$$\phi = 2\pi\, r\, f. \tag{5b}$$

Then, by the Calculus of Variations (Ref 4), the form of f that will make A_L a maximum subject to Equation 5 is obtained by solving the following equation:

$$\frac{\partial F}{\partial f} + \lambda \frac{\partial \phi}{\partial f} = 0 \qquad (6)$$

where λ is some constant that can be determined from the constraining condition (Equation 5).

17. Now, from Equations 5a and 5b, Equation 6 can be rewritten as

$$2\pi r\left[\sec\{\tan^{-1} r/h\}\, g\, p \exp\left(-f \sec\{\tan^{-1} r/h\}\, g\, p\right) + \lambda\right] = 0 \qquad (7)$$

It follows that

$$f = \frac{\ln\left[\sec\{\tan^{-1} r/h\}\, g\, p\right] - \ln(-\lambda)}{\sec\{\tan^{-1} r/h\}\, g\, p} \qquad (8)$$

18. The constant λ is now determined from Equations 8 and 5. Thus,

$$-\ln(-\lambda) = \frac{N/2\pi - \int_0^{r_0} \frac{\ln\left[\sec\{\tan^{-1} r/h\}\, g\, p\right] r\, dr}{\sec\{\tan^{-1} r/h\}\, g\, p}}{\int_0^{r_0} \frac{r\, dr}{\sec\{\tan^{-1} r/h\}\, g\, p}} \qquad (9)$$

Hence, combining Equations 8 and 9, the form of f that is optimum is

$$f = \frac{\ln\left[\sec\{\tan^{-1} r/h\}\, g\, p\right]}{\sec\{\tan^{-1} r/h\}\, g\, p} +$$

$$\frac{N/2\pi - \int_0^{r_0} \frac{\ln\left[\sec\{\tan^{-1} r/h\}\, g\, p\right]}{\sec\{\tan^{-1} r/h\}\, g\, p}\, r\, dr}{\sec\{\tan^{-1} r/h\}\, g\, p \int_0^{r_0} \frac{r\, dr}{\sec\{\tan^{-1} r/h\}\, g\, p}} \qquad (10)$$

19. The above expression for f is not yet final, however. There is still

the requirement that f be always non-negative. In this connection, the condition of non-negativity from Equation 8 is

$$\sec\{\tan^{-1} r/h\}\, g\, p \geq (-\lambda) \qquad (11)$$

20. The above inequality sets the limits of integration in Equation 10. These limits are computed as follows: Let the integrals in the numerator and denominator of Equation 9 be denoted by $I_1\{-\lambda\}$ and $I_2\{-\lambda\}$, respectively, to indicate the functional dependence of the integrals on $(-\lambda)$. Then Equation 9 can be rewritten as

$$N/2\pi = I_1\{-\lambda\} - \ln(-\lambda)\, I_2\{-\lambda\} \qquad (12)$$

21. Now construct a plot of $\sec\{\tan^{-1} r/h\}\, g p$ Vs r from the particular set of data at hand. There will be a certain range corresponding to the dependent variable's axis over which $-\lambda$ can take on values satisfying the inequality stated in Equation 11. The requirement is then to select from that range the particular $-\lambda$ that satisfies Equation 12. (In practice, about 3 or 4 tries will usually suffice for this.) The correct limits of integration are then simply read off the r axis of the graph.

22. From Equation 10, the final form of f, letting $\tan^{-1} r/h$ equal a, is

$$f = \frac{\ln[(\sec a)\, g\, p]}{(\sec a)\, g\, p} + \frac{N/2\pi - \displaystyle\int_{(\sec a)\, g\, p > -\lambda} \frac{\ln[(\sec a)\, g\, p]}{(\sec a)\, g\, p}\, r\, d\, r}{(\sec a)\, g\, p \displaystyle\int_{(\sec a)\, g\, p > -\lambda} \frac{r\, d\, r}{(\sec a)\, g\, p}} \qquad (13)$$

23. It turns out that the expression for f given by Equation 13 can be easily manage in numerical work.

24. A somewhat unexpected result that follows from the foregoing analysis is that, in the process of finding the optimum distribution of fragments, the cone angle for those fragments is automatically determined. The fact that the optimum distribution has certain required limits of

integration (Equation 13) shows this plainly. Consequently, any pre-established cone angle (for example, the 156° based on terrain considerations) could easily prove to be inappropriate for any number of particular situations.

25. Terrain considerations will, however, establish an upper bound for the cone angle, in which case we have

$$0 < 2\alpha < 156^\circ \qquad (14)$$

where α is the cone angle given in Equation 13.

26. Whether the upper bound is the 156° of Equation 14 some other angle will depend on the advances made in terrain studies.

DISCUSSION OF CURVES

27. This report contains a calculation of the lethality of three air-ground warhead contours *under the following specific conditions:*

a. There is a total of 7080 fragments on the surface of each warhead.

b. Each fragment is a 20.6 grain cube.

c. The initial velocity for all fragments is 3340 feet per second.

d. Each of the three warheads bursts statically at a height of 30 feet and at an angle normal to the ground plane.

e. The probability that a hit by a single fragment will incapacitate is based on the Type B disablement curve of BRL (Ref 5).

f. The average presented area of a human target is given by $g = 3.4 + 1.1 \cos \alpha$ (Ref 1). (The foregoing conditions have purposely been made arbitrary to avoid involvement with a higher security classification than is warranted by the objectives of this report).

28. The description is not intended to serve as a basis for generalization, but is included only to make a single point clear; namely, that when the ground distribution of fragments deviates from some theoretical optimum, losses are incurred in lethal area.

29. In Figure 1 (p 10) density of fragment distribution on the ground is

plotted against distance from ground-zero for the three warhead contours. Fragment density is represented by the function f (r), in fragments per square foot. Distance from ground-zero is represented by the variable "r", in linear feet.

30. Given a prescribed area-target on the ground (in the case illustrated by Figure 1 (p 10) this is a circle with a radius of about 141 feet), the graph shows how a given number of fragments (7,080) emanating from a height of about 30 feet are distributed within that area. The graph shows plainly that each warhead shape[1] gives a different ground distribution of the fragments.

31. The contour that is least efficient lethally, for the conditions given, is the sphere (A_L = 16,500 ft^2). One reason, suggested by Figure 1, is that this contour places far too many fragments in, roughly, the inner half of the target area and far too few in the outer half. The results are thus either relative "over-killing" or "under-killing".

32. Figure 1 further shows that the Kent-Hitchcock contour is greatly superior (A_L = 22,100 ft^2) to the sphere in the above sense.[2] However, a comparison with the theoretically optimum contour, denoted ERS-Optimum (A_L = 23,600 ft^2), indicates that the Kent-Hitchcock contour, in spreading a uniform density of fragments throughout the target area, leads to essentially the same kind of inefficiency as the sphere, though to a lesser extent. There is a margin of relative under-killing for most of the target area, and on the periphery there is a relative over-killing.[3]

33. Figure 2 (p 11) is a graph (for the same conditions and contours as Figure 1) of probability of kill, P_k, against distance from ground-zero, r. It is included for the purpose of showing the correspondence between P_k and density of fragments f (r).

---

[1]The warhead shape corresponding to the ERS-Optimum has not been specified except in terms of the ground distribution. In the present work, this is not pertinent. However, it is planned to find the corresponding warhead shape in future work.

[2]It is not always true that the Kent-Hitchcock Contour gives a more efficient distribution of fragments than a sphere.

[3]The superiority of the Optimum distribution over the Kent-Hitchcock in this example is rather slight. For certain conditions that margin will be much greater; for other conditions, it will be less. In either case, the principle is still the same. Deviations from the optimum pattern lead to relative under-killing and over-killing.

ACKNOWLEDGEMENT

Appreciation is expressed to Dr. Sylvain Ehrenfeld for his many helpful suggestions and criticisms.

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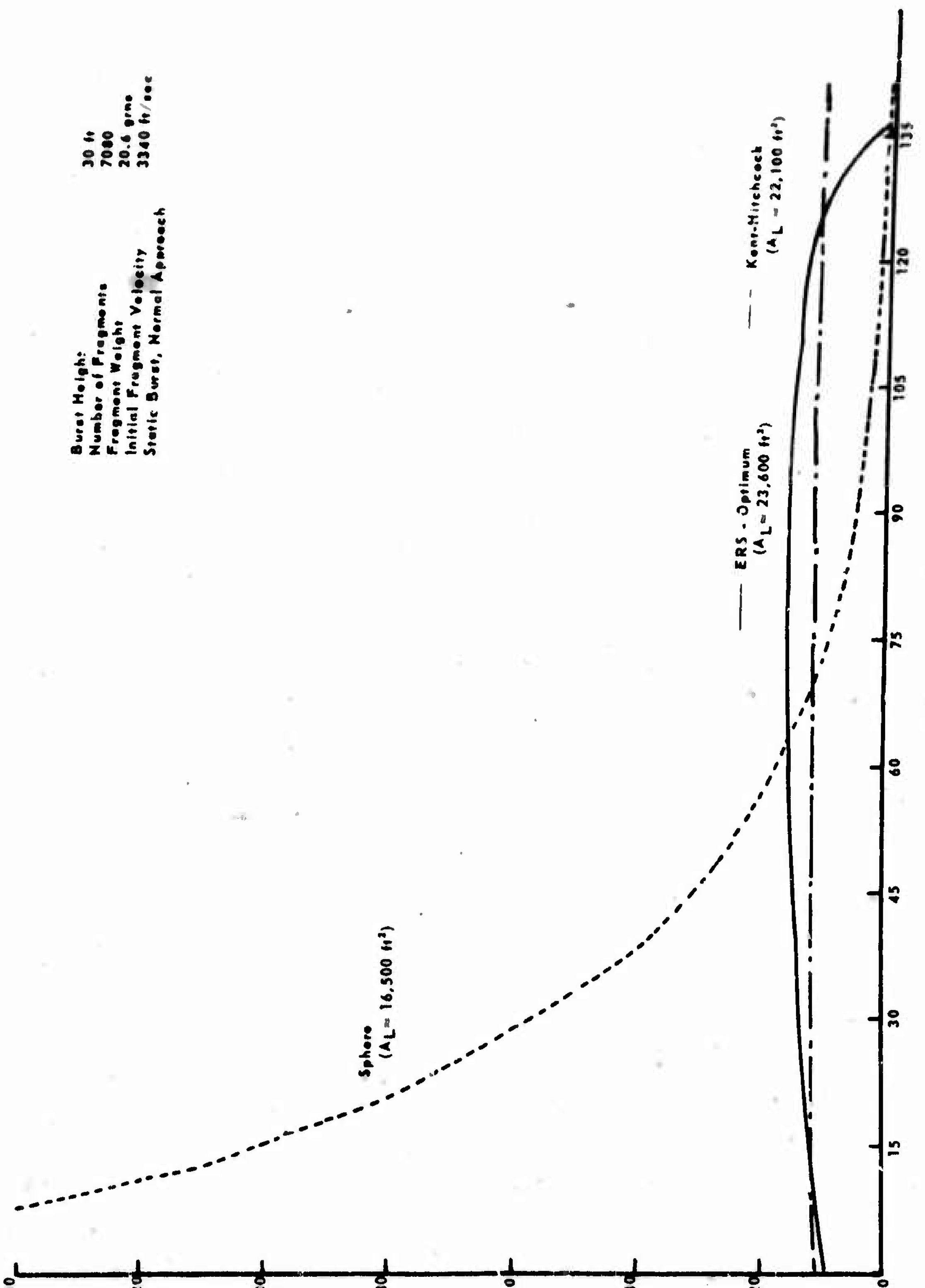

Fig 1 Fragment Density on Ground vs Distance from Ground Zero

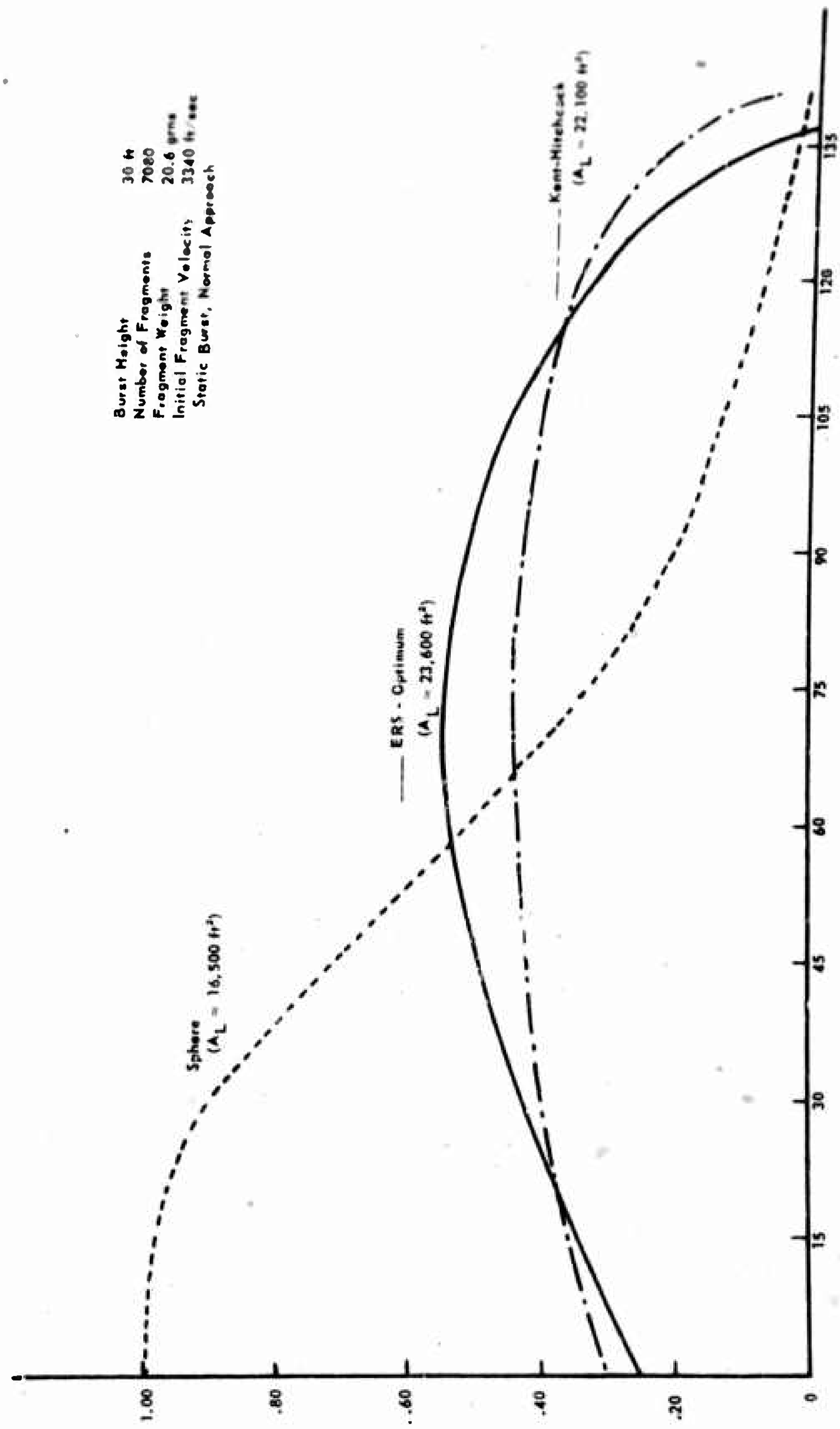

Fig 2 Probability of Kill vs Distance from Ground Zero

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